DONALD L. LIGHT*
U.S. Army TOPOCOM
Washington, D.C. 20315

Photo Geodesy from Apollo

The exploitation of the sensors of Apollo 15, 16 and 17 will mark a new era in the use of observations from the related disciplines of photogrammetry geodesy and astrodynamics.

ABSTRACT: *NASA's Apollo 15, 16 and 17 Missions are advancing the state-of-the-art over previous lunar camera systems by orbiting an advanced metric mapping camera system around the moon. This stellar oriented mapping camera system includes an associated panoramic camera, laser altimeter and other data sensors. The planned lunar photographic coverage is indicated. Concepts planned for reducing the data include dynamical methods of lunar triangulation with orbital constraints. Particular attention is focused on the aspects of developing a lunar control network, gravity-model improvements, and an improved figure of the moon, along with the usual mapping and orthophoto productions. The exploitation of these advanced data sensors marks a new era in the utilization of observations from the related disciplines of photogrammetry, geodesy and astrodynamics all coupled with computer science.*

INTRODUCTION

FOR CENTURIES man's history and exploration of the earth has been recorded best on maps and charts. The more representative and accurate the maps and charts, the better the exploration was documented. A similar situation seems also true for the moon. Positional knowledge and descriptions of the moon and its surface features are particularly vital to the disciplines of geology, geophysics, and astronomy and is therefore of priority interest to the mapping profession which, since the dawn of history, have supplied these data to mankind. NASA's Lunar Orbiter and Apollo Programs through Apollo 14 have produced phenomenal photography to support landing-site selection and surface operations. However, the new stellar-oriented, metric-mapping camera system and its laser altimeter and panoramic camera, which have been planned for the Apollos 15, 16 and 17, offers an order of magnitude improvement toward producing modern contributions to lunar scientific objectives. This paper describes and illustrates this system and its associated scientific applications, with emphasis on modern concepts of photogrammetric/geodetic satellite geodesy.

In 1965 and 1967, NASA convened conferences on Lunar Exploration and Science[1,2] at which distinguished scientists from the disciplines of geology, geophysics, and geodesy/cartography recommended scientific objectives and a mapping program which could be addressed by an orbital metric mapping camera system. These scientific objectives were:

- Refine the lunar ephemeris describing the position of the moon's center of mass with respect to the earth.
- Refine the physical librations of the moon about its center of mass.
- Define a mathematical reference surface and coordinate system with origin at the center of mass, and axes aligned with the principal axes of inertia.
- Determine a coordinate network of photo-identifiable control points on the lunar surface with an accuracy of 10 to 15 meters.
- Provide a mathematical expression for the spatial variations in the moon's gravity field.

In addition several map series were recommended to support geologic planning, mission operation and post-mission analysis. The

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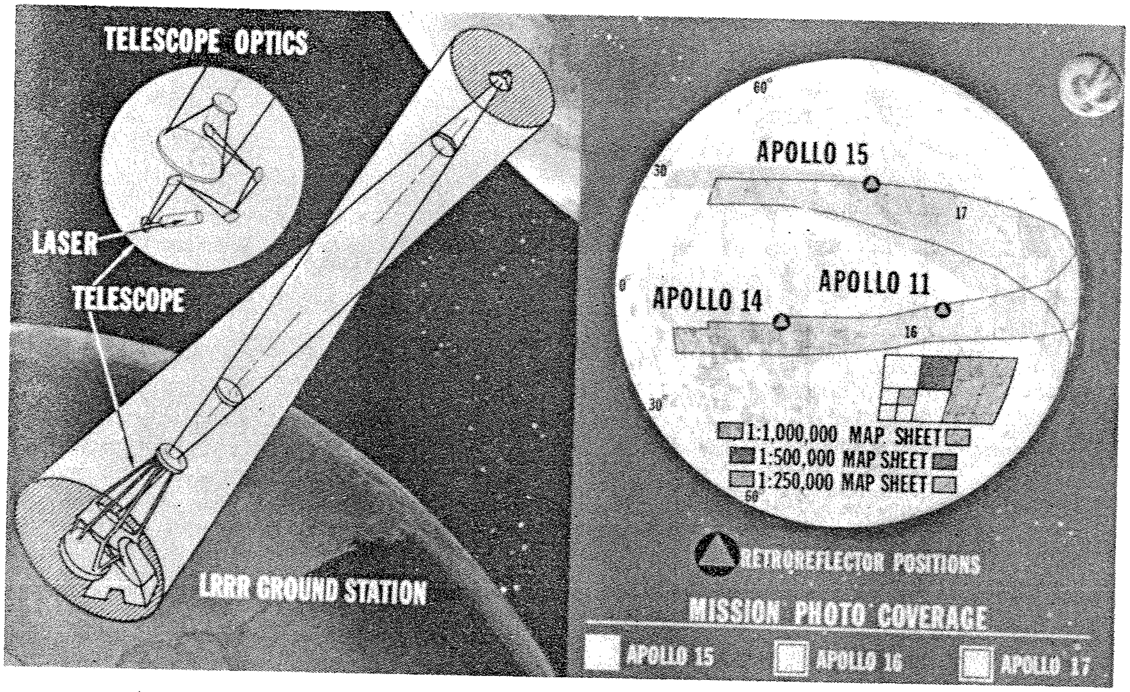

FIG. 1. Apollo photo coverage.

affects of budget cuts in the Apollo program may prolong the accomplishment of some of these objectives. It is anticipated that the mapping program will consist of topographic line and photo maps and mosaics ranging in scale from 1:25,000 to 1:500,000. Some investigators are interested in small-scale mosaics at scales of 1:1,000,000 for regional analysis. Emphasis will probably be placed on supporting the needs of the various principal investigators and their scientific experiments; this may require a total mapping effort.

To pursue these objectives, in October of 1969 NASA formed the "Apollo Orbital Science Photographic Team" to serve as scientific advisors for maximizing the scientific return from the remaining missions. The team, whose chairman is Mr. Frederick J. Doyle, is composed of 12 representatives from universities, industry, NASA, and governmental mapping agencies. The team serves as advisors to the Orbital Science and Geodesy/Cartography Programs for camera instrumentation design, mission planning, data reduction and analysis.

Potential landing sites and typical mapping photographic coverage for the three missions are illustrated in Figure 1. Apollo 15, the highest inclination mission, is called Hadley-Apennine. The site is about 3° west of the center of the moon at about 25° north latitude or 465 miles north of the lunar equator. Mission 16 is planned for the Descartes area. Higher, near polar, orbital inclinations are desirable for selenodetic science applications, but are not planned under the present program.

MAPPING CAMERA SYSTEM

With the 3-inch metric Mapping Camera System (MCS) aboard the Apollo spacecraft for Apollos 15, 16, and 17, photogrammetric geodesy takes on an intriguing new dimension. The MCS, built by Fairchild, will be mounted in the front shelf of the Scientific Instrument Module (SIM). Figure 2 illustrates the SIM. The MCS is composed of a terrain camera with an associated stellar camera, laser altimeter, and precise timing mechanism. The mapping camera will photograph the lunar surface while the stellar camera simultaneously looks to the side of the orbital plane and about 4° to 8° above the horizon for the purpose of providing starfield photos for the reduction of the attitude angles of the stellar camera. The dual camera unit is shown in Figure 3.

The stellar camera's attitude angles are later related to the mapping camera which provides orientation of the mapping camera with respect to the lunar surface based on the right ascension, declination system of the stars. The stellar camera is expected to record from 25 to 75 star images depending on the location in the celestial sphere. The laser altimeter is synchronized to fire simultaneously with the MCS and is aligned parallel with the optical axis of the MCS. The output

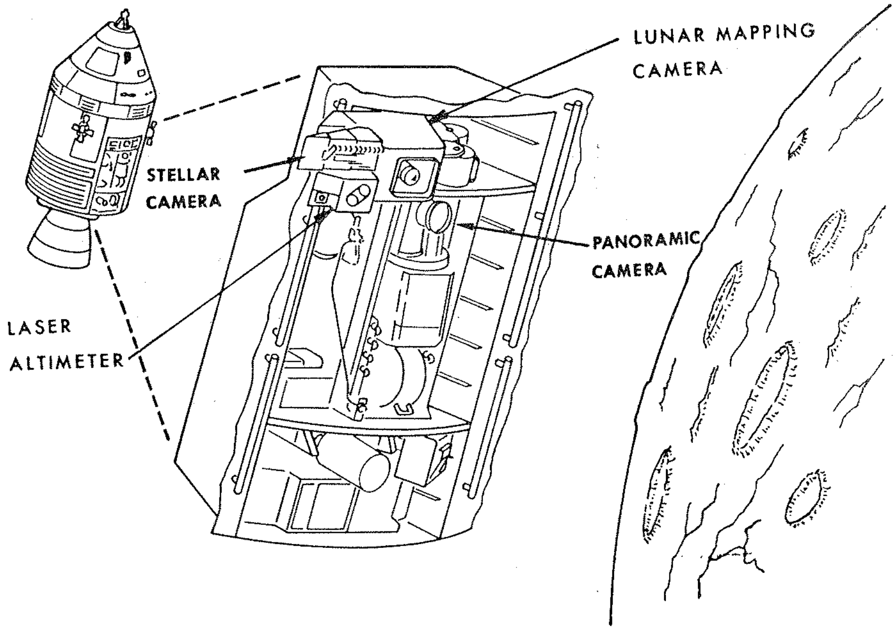

FIG. 2. Scientific instrument module with the selodetic science cameras deployed.

is a measured distance from the spacecraft to the lunar ground for each photo given, with a precision of ± 2 meters. The altimeter is built by RCA. The time of each exposure is given to one millisecond. Film deformation can be minimized by virtue of the 10-mm reseau engraved on the glass focal-plane plate. For the nominal altitude of 111 km (60 nautical miles) with Kodak 3400 film, the expected resolution will range from about 30 to 60 meters per line-pair depending on the solar altitude. Details of the system are well documented by Doyle[3] and are only summarized here for background to the applications. Figure 4 illustrates the total system as configured in orbit as a data collection system and Table 1 gives its characteristics.

APOLLO OPTICAL-BAR PANORAMIC CAMERA

Another very important item in the SIM bay is the 24-inch Optical Bar Pan-Camera built by Itek. It will provide very high-resolution photographs of most of the surface

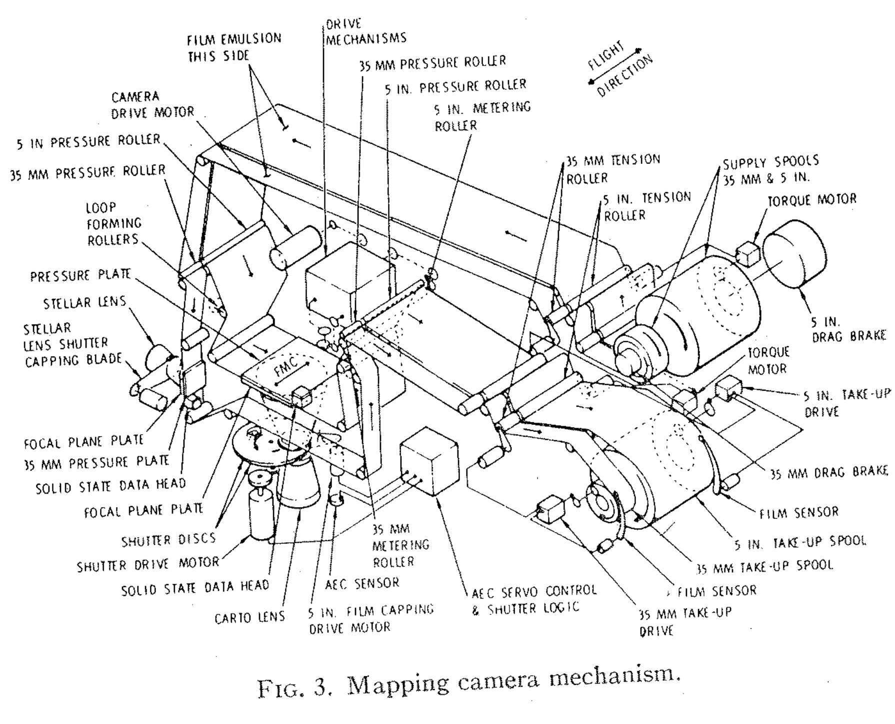

FIG. 3. Mapping camera mechanism.

TABLE 1. SUMMARY APOLLO SIM CAMERA CHARACTERISTICS

Characteristics	*Mapping Camera System* *Terrain*	*Stellar*	*Terrain Analysis*
Manufacturer	Fairchild camera & Instrument Corp.	Fairchild camera & Instrument Corp.	Itek-Optical Systems Division
Basic design	Frame	Frame	Panoramic
Focal length	3 inches (76.6 mm)	3 inches (76 mm)	24 inches (610 mm)
Lens angular coverage	74°×74°	24°×18°	10°46′×108°
Lens aperture	f/4.5	f/2.8	f/3.5
Lens distortion	<50μm	<10μm	—
Filter	None	None	None
Resolution lp/mm	@1.7:1, 58–70	To star magnitude 6	@2:1, 108–135
Reseau-interval	10 mm crosses (121)	5 mm crosses (22)	—
Illumination	Natural	Artificial	—
Fiducials—Artificial	2 sets of 4	4	Marks
Natural	4	None	—
Shutter Type	Between Lens	Between Lens	Focal Plane-variable slit
Shutter speeds	1/15 to 1/240 sec.	1.5 sec-fixed	1/80 to 1/500 sec.
Format	4.5×4.5 in.	0.88×1.25 in	4.5×45.25 in
Transmission	44.3%	93%	55–78%
Film size and typical type	5 inches (127 mm) Type EK 3400 or EK 3414	1.38 inches (35 mm) (Non Perf.) Type 3401	5 inches (127 mm) Type EK 3414
Magazine	1500 feet	510 feet	6500 feet
Capacity	(3600 frames)	(3600 frames)	(1617 frames)
Cycling rate	8.5 to 34 sec.	8.5 to 34 sec.	4.96 to 16.95 sec.
Motion compensation	10 to 40 M Radians/sec.	None	10 to 20 M Radians/sec.
Exposure control	16:1 Automatic	Fixed (1.5 sec.)	Automatic plus bias tuning
Weight	131 Lbs W/Film	Total	343 Lbs W/Film
Data Recording	Data Block	Time; Serial No.	GMT; Frame INCR; V/h in MR/sec.
Film Flattening	Platen with moveable pressure plate	Platen with moveable pressure plate	Rollers
Stereo rock	—	—	$\pm 12\frac{1}{2}°$ plus IMC

covered by the frame camera. In general, it will not expose simultaneously with the mapping camera. From the nominal altitude of 111 km, the pan camera is expected to provide about 2 meters surface resolution at the spacecraft nadir.

In the stereo mode as illustrated by Figures 5 and 6, the pan camera rocks through the 25° convergence angle between each two exposures. The forward exposure from station 1 and the aft exposure from station 6 will overlap by 100 percent, as shown by Figure 6, to form a stereo model. Each succeeding stereo model will overlap the preceding one by 10 percent. The unique geometry of the optical bar concept is illustrated in Figure 7. Doyle[3] gives further details of the pan camera. The total system integration into the SIM is performed by the North American Rockwell firm. The important components of the total system as viewed by the photogrammetrist are illustrated in Figure 4. Table 1 provides a summary of its important characteristics.

Doppler tracking of the spacecraft by NASA's Tracking Network will provide orbital elements, ephemeris position and velocity vectors for each photo exposure. The nominal speed of the spacecraft is 1627 m/sec based on the 111 km circular orbit. By virtue of the 0.001-second timing for each exposure, relative distances between exposures will be known to ±2.3 meters.

Several factors contribute toward making this system much improved over previous lunar mapping systems. These factors include:

* Flight film returned to earth.
* Higher photograph resolution.
* Reseau in the camera to control film deformations.
* Attitude constraints for each photo obtained from stellar photography.
* Precise timing of exposure station positions related to orbital ephemeris data.
* Laser altimeter distance from spacecraft to ground for each photo.

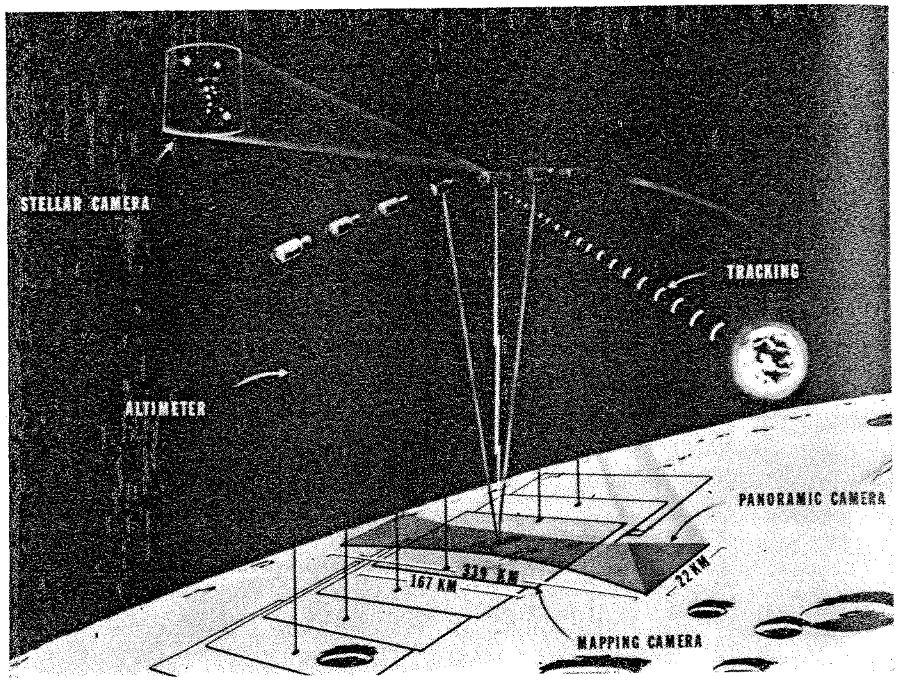

FIG. 4. Apollo mapping camera systems.

APPLICATIONS AND METHODS

Several scientific applications have been initiated which plan to utilize the data collected with the Apollo data sensors. Esenwein, Roberson and Winterhalter of NASA's Lunar Exploration Office have noted all of these in Reference 4. One application of primary interest to many lunar scientists is the establishment of a lunar control-point reference system, and its associated selenodetic constants. Establishing a well documented and accurate reference system will provide a basis for a total mapping program. A synthesis of data from the landing sites into an overall picture of events, such as would be possible through a mapping program, would increase the scope and strengthen the systematic approach to understanding the moon's origin and evolution. A broad view of lunar processes may suggest forces which have controlled lunar evolution.

The control-point triangulation problem is most interesting to a photogrammetric geodesist for it gives him an opportunity to exploit data heretofore unavailable. The triangulation problem is interrelated with each of the other selenodetic applications, besides being fundamental to a sound mapping program. Rigorous, computational methods of analytical photogrammetry as introduced in this country by Dr. Hellmut Schmid[5] and Duane Brown[6] will play a major role. The MCS provides a great deal of auxiliary information about the photogrammetric parameters, and it is necessary to incorporate these data into observation equations and then impose a least squares criterion. The preferred method is that the parameters and observations enter as constrained variables with appropriate statistical estimates of each parameter and observation to be used as weights. Appropriate geometric and dynamic constraints are imposed on the parameters and reduced in a simultaneous system.

To begin the photogrammetric triangulation, one needs a post-flight ephemeris as reduced primarily from the doppler tracking data. This ephemeris will be computed by NASA using a program called *Houston Operations Predictor/Estimator* (HOPE).[7] The HOPE Program accepts, as input, the classical observations, which include range, range rate, azimuth, elevation, radar X, Y-angles, hour angle, declination and doppler.

In addition to the HOPE program for tracking data reduction, there is a companion program called *Apollo Photograph Evaluation* (APE). Together, these two programs produce initial estimates of the position history of the spacecraft for each exposure, rough estimates of the orientation of the frame, pan, and stellar cameras and the predicted star field in the stellar camera's field of view, as well as a cursory reduction of the altimeter's slant range. These parameters are invaluable as initial inputs to the follow-on triangulation work which will combine both tracking and

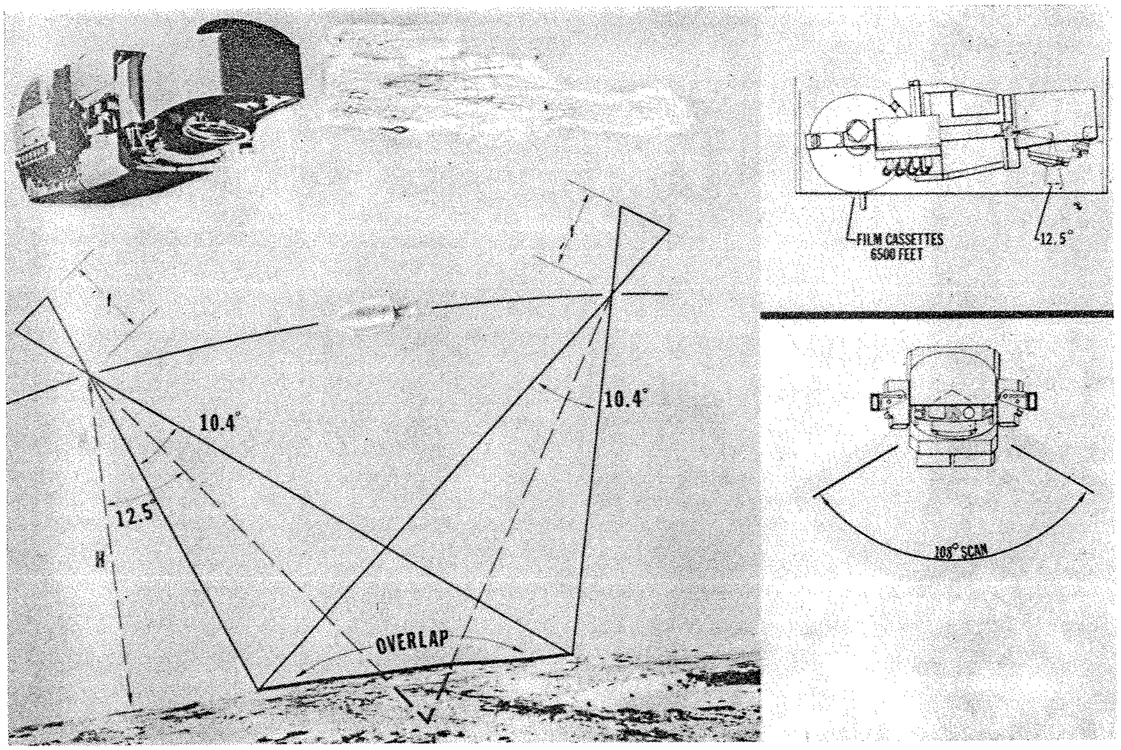

FIG. 5. Apollo optical-bar panoramic camera. Focal length, 24 inches; $f/3.5$; film, 5 inches wide; format, $4\frac{1}{2}$ by 45 inches; 1,650 exposures.

laser altimeter distance in a simultaneous least-squares adjustment.

Camera Attitude Orientation Elements

The stellar camera is the sensor that links the mapping system to the celestial coordinate system. Attitude determination accurate to ±15–30 sec of arc for the three orientation angles is a practical expectation. The procedure is well documented in the Manual of Photogrammetry, Chapter IV by Sewell,[8] but is summarized here for completeness.

STELLAR CAMERA ORIENTATION IN CELESTIAL COORDINATE SYSTEM

- ⋆ Identify stars on film.
- ⋆ Update star right-ascension and declination coordinates.
- ⋆ Measure star coordinates (x, y) on stellar camera plates.

The relationship between the two coordinate systems is then a 3 by 3 orthogonal rotation matrix:

$$\underset{3\times1}{x_s} = \underset{3\times3}{M_s}\ \underset{3\times1}{\alpha}$$

where x_s represents the directions of measured image coordinates in the stellar camera system, α the direction cosines of updated stars in the celestial coordinate system, and M_s is the orientation matrix of the stellar camera in the celestial coordinate system. Now a matrix multiplication provides the orientation of the mapping (terrain) camera in the celestial system:

$$\underset{3\times3}{M_t} = \underset{3\times3}{M_{st}}\ \underset{3\times3}{M_s}$$

where M_s is the stellar camera orientation matrix in the celestial system, M_{st} is the precalibrated relative orientation *lock-angle* ma-

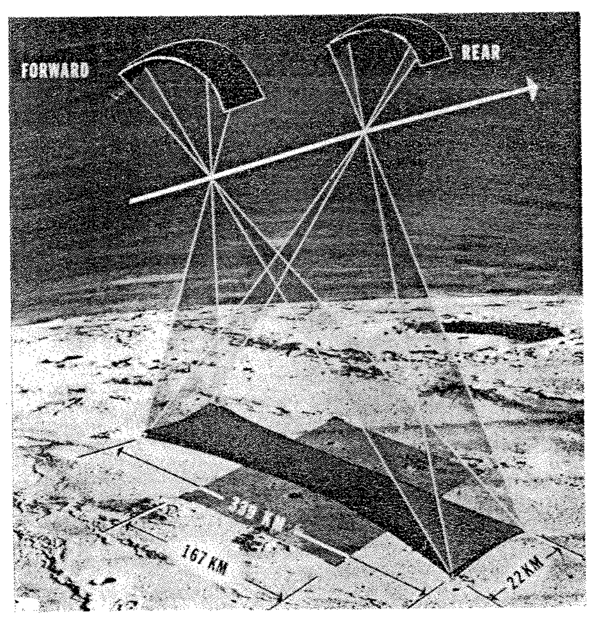

FIG. 6. Panoramic stereo model.

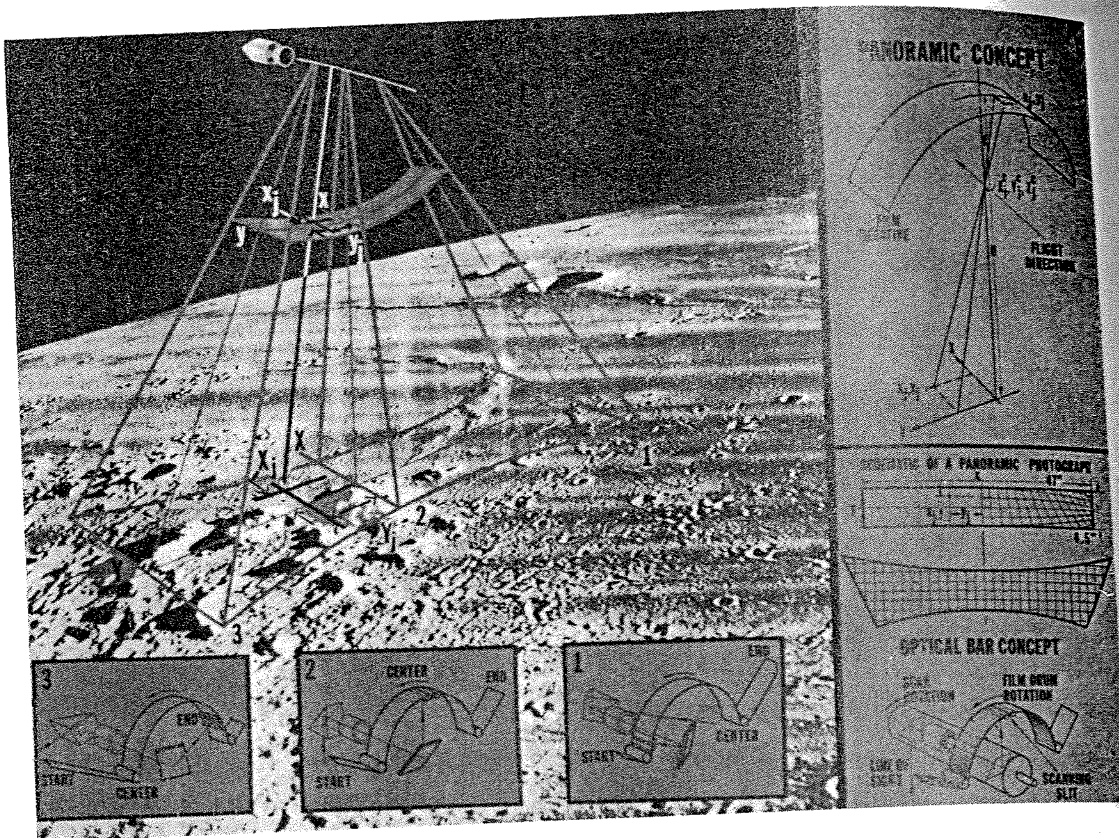

FIG. 7. Panoramic geometry.

trix between stellar and terrain cameras, and M_t is the orientation matrix of the terrain camera in the celestial system. The orientation matrix M_{ct} of the terrain camera in the selenocentric coordinate system is really the objective. Therefore M_{ct} is given by a final rotation

$$\underset{3\times 3}{M_{ct}} = \underset{3\times 3}{M}\ \underset{3\times 3}{M_t}$$

where M_{ct} is the terrain-camera orientation in selenocentric coordinate system, and M is the relative orientation matrix between selenocentric moon system and celestial $(\alpha,\ \delta)$ coordinate system.

The moon's orientation M in the celestial (earth centered) sphere and its relationship to the earth-based Apollo trackers presents some interesting problems, in addition to providing the connecting link for attitude constraints.

Resolution of the lunar surface obtainable from earth observations restricts knowledge of the moon's axial orientation in the celestial sphere.

The moon's orientation M can be described in terms of three angles as given by Baker:[12]

$$M = f(\gamma, i, \Omega').$$

These angles are themselves functions of other time-dependent parameters of extreme interest, that is:

* Moon's orientation in celestial sphere,
* Moon's rotation rate, and
* Moon's physical librations (ρ, σ, τ).

Accurate knowledge of these angles is important to model accurately the moon's motion. The librations are tabulated in the *American Ephemeris and Nautical Almanac* to only $0°.01$ as a consequence of the uncertainties arising out of the observational data. This $0°.01$ represents 300 meters distance on the moon's surface. The popular physical libration models in use today are by Hayn, Koziel, and Eckhardt. Physical librations occur in both longitude and latitude.

Discussion with my colleague, Jesse B. Schreiter,[9] relative to his libration model experiments at TOPOCOM, has shown that, upon using the same input data for three different models, the resulting lunar surface positions may differ by as much as 300 meters. This verifies that the libration coefficients are not accurately known. Figure 8, compiled by Schreiter, illustrates that the physical libration in longitude has a period of one year, and the physical libration in latitude has a period of approximately six years. Some of the inconsistency in the ephemerides com-

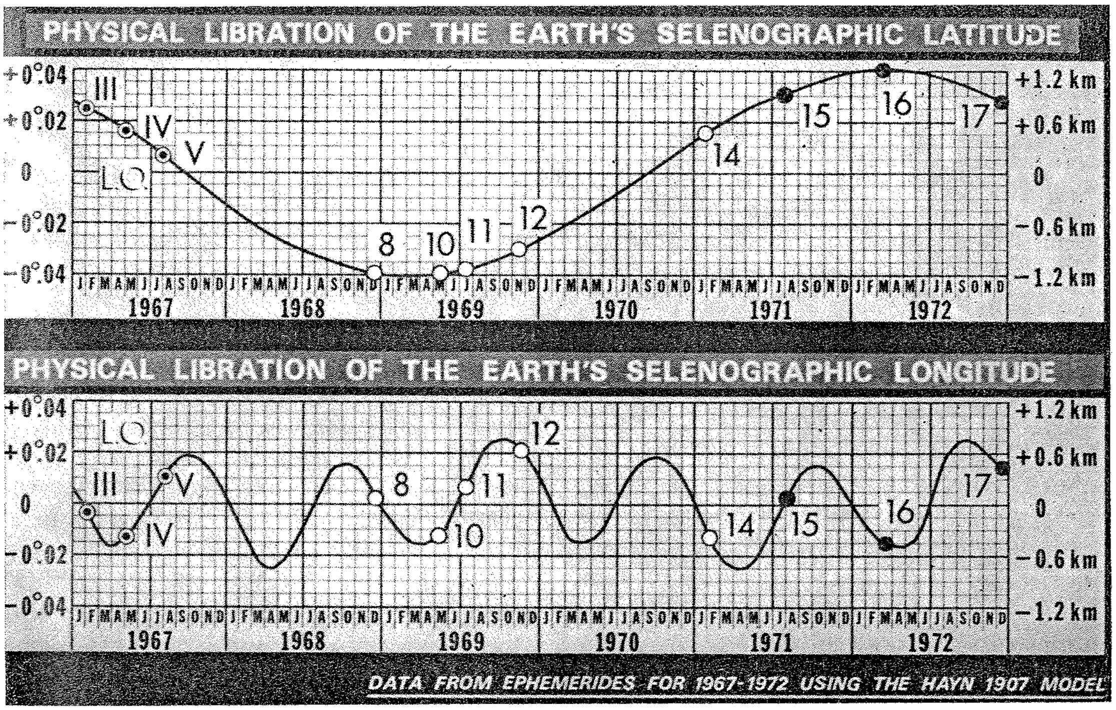

FIG. 8. Physical librations of the Moon expressed in selenographic coordinates.

puted for NASA's previous five Lunar Orbiter satellites can be attributed to the inaccuracy of these libration angles.

Experience with Lunar Orbiter triangulation showed that unless these values can be improved, or differentially corrected in some manner, directional accuracy from the stars via the stellar camera cannot be fully exploited in a moon-centered system. Notice on Figure 8 that the three points that represent the time of missions 15, 16, and 17 do not adequately describe the libration curves. Observations spread over a 3- to 6-year period are needed to define fully and, therefore, improve the coefficients of the libration models. Combining the tracking data from the Lunar Orbiter (L.O.) Missions IV and V with the lunar Apollos as plotted on Figure 8 would describe these librations over a time span sufficient to attempt a refinement of the libration model. L.O. IV and V were polar orbits, at altitudes of 2,700 and 100 km, respectively. However, the combined analytical solutions as mentioned earlier may offer a basis for the differential correction of the libration parameters so that consistency between common photographic image points taken from different missions seems possible.

The photogrammetric Equations 1 are independent of the nature of the motion of the Apollo vehicle. For example, the exposure station could, in principle, be completely randomly distributed in space. This is the geometrical mode as in MUSAT and is shown by Equation 1.

But, if we wish to exploit the fact that during each orbital pass, the spacecraft proceeds along a free flight orbital trajectory and all photos lie on this orbital arc, we must introduce orbital constraints. From astrodynamics (Ref. 12), the camera stations X^c, Y^c, Z^c can be given as functions of the six Keplerian orbital elements $(a, e, i, \Omega, \omega, t_i)$ or as position and velocity components $(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})$ at a given time t_i. In other words, spacecraft position at some arbitrary epoch t_0 may in principle be expressed as:

$$\begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix} = g[X_0, Y_0, Z_0, \dot{X}_0, \dot{Y}_0, \dot{Z}_0, t_0, C_{nm}, S_m, \text{other constants}] \quad (2)$$

Substituting Equation 2 into Equation 1, we have the basic condition equation for the Apollo LOSAT/LOBAT Program as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = F_2[(\phi, \omega, \kappa, t)_i, (X_0, Y_0, Z_0, \dot{X}_0, \dot{Y}_0, \dot{Z}_0, C_{nm}, S_{nm}), (X, Y, Z)_j]. \quad (3)$$

Here x_p, y_p, f are usually given as constants and are left out of Equation 3 for simplicity. C_{nm}, S_{nm} are given coefficients of the lunar gravity potential function and are constants in LOSAT/LOBAT.

The laser altimeter distance is expected to contribute a strong scale constraint to the triangulation equations. Reference 13 discusses the geometry of the altimeter distance in detail. Previous lunar orbiters did not have either the altimeter or the stellar camera information. As the altimeter and the metric camera are aligned and they expose simultaneously, the image coordinates of the spot to which the altitude is measured will be known. This point's ground position will be computed in the photogrammetric solution, and the measured distance D provides a condition equation between that point and the corresponding camera station:

$$D = [(X^c - X^a)^2 + (Y^c - Y^a)^2 + (Z^c - Z^a)^2]^{1/2} \quad (4)$$

where D is the measured distance at time t, X^c, Y^c, Z^c are the coordinates of spacecraft at time t, and X^a, Y^a, Z^a are the coordinates of illuminated point on surface.

Figure 9 illustrates the usage of the altimeter's terminus point as a constraint. Each altimeter terminus point on the surface will be viewed on the other 8 photos surrounding the one of interest. For the moment, consider a group of 9 photos as a unit with the 8 conjugate image points being constrained to the center picture's distance D. Then expand the concept as is shown with the 3 units and 27 photographs. It is readily apparent that the altimeter provides vital scale constraint data. Its total contribution will be of interest to numerous experiments.

The M_{ct} matrix can be heavily weighted in the solution to enforce the attitude of the terrain mapping camera, because this is considered to be very accurate information. Consequently, differential corrections would be appropriately applied to the libration parameters and a unified control point network would be possible. Ranging retro-reflectors are located at the sites of Apollos 11, 14 and 15 and they should be identifiable in the photography. Current ranging accuracies (Earth to Moon) are on the order of ±15 centimeters, and this accuracy will be reflected in the accuracy of the three-point positions. Figure 1 shows the triangle formed by these points which could serve as an absolute datum for the lunar reference surface. NASA's Laser Retro-reflector Experiment, is in progress, and it is a promising means for improving the libration models, and should be able to collect data over a six-year period.

Strip and Block Triangulation

The mapping camera has a capacity of 3600 frames per mission with about 2100 being typical for use in the triangulation reduction. The flight plan indicates that some of the strips may be up to 230 photos each. Basically there are two approaches. One is purely geometric and the other imposes dynamical constraints. That is, the camera stations are constrained to fit on the orbit as reduced from the tracking data. The geometrical solution can be accomplished by the well known Multiple Station Analytical Triangulation Program (MUSAT)[10] and the dynamical solution by an Apollo version of the Lunar Orbiter Strip/Block Triangulation (LOSAT/LOBAT).[11]

The Apollo version is presently under development for NASA's Cartography Office by

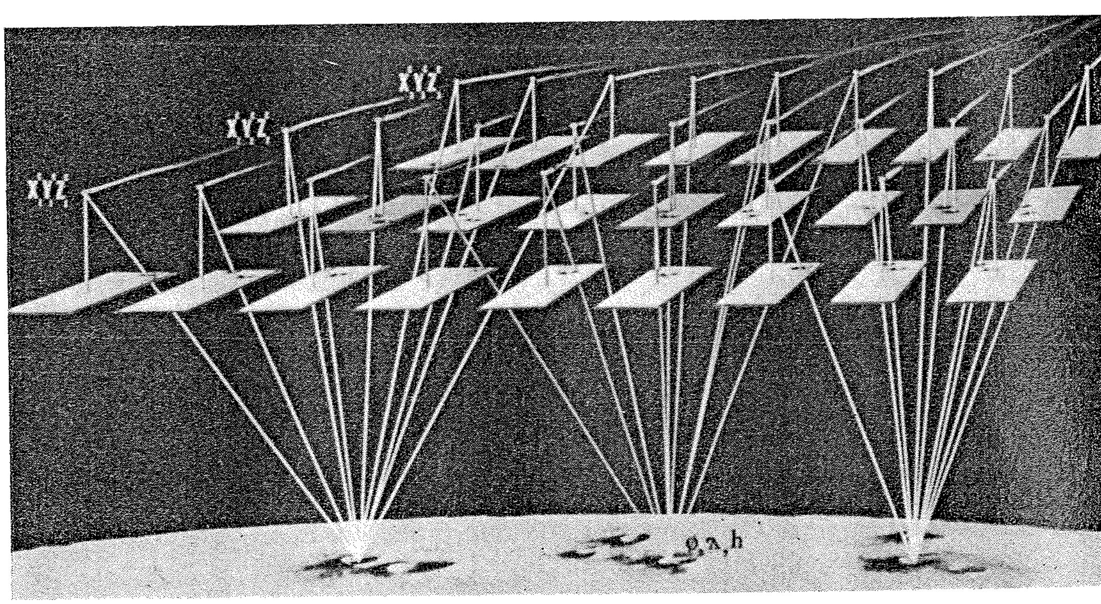

FIG. 9. Multi-photo ray concept with altimeter constraints.

DBA Systems under the direction of the Army Topographic Command and the Aeronautical Chart and Information Center. Each of these programs are uniquely outstanding where they apply, but they do have limitations where we wish to approach the problem on a grand simultaneous adjustment basis. By grand adjustment, we mean computing ground positions of image points, camera stations and gravity model coefficients as one iterative problem. A third program called TRACE 8 *Apollo* has been proposed for orbital ephemeris, gravity model and libration computation. The program is to work in concert with MUSAT and/or Apollo LOSAT/LOBAT to solve the ephemeris, gravity model, libration model and the control point problems in an iterative manner.

The basic relationship for the two photogrammetric programs is the pair of fundamental projective equations of analytical photogrammetry. These equations are given by Schmid[5] and Brown[6] and can be functionally represented as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = F_1[x_p, y_p, f, (\phi, \omega, \kappa), (X^c, Y^c, Z^c), (X, Y, Z)] \quad (1)$$

where x, y are the photo image space coordinates of the projected object space point from the lunar surface, x_p, y_p are the image space coordinates of the principal point, f is the principal distance or focal length of camera, ϕ, ω, κ are the orientation angles of photo in the selenocentric system, X^c, Y^c, Z^c are the object space coordinates of each photo on the orbit and X, Y, Z are the object space coordinates of the lunar ground object.

Now that we have considered the contribution of each of the sensors as illustrated in Figure 4 and represented it by an equation, it seems appropriate to view these as a simultaneous system of observations ready for adjustment by the method of least squares. Writing each set of observation equations in linearized form, we represent, from Figure 4:

Photogrammetric equations—

$$V + \ddot{B}\ddot{\delta} + \overset{\circ\circ}{B}\overset{\circ\circ}{\delta} + \overset{\cdots\cdot}{B}\overset{\cdots\cdot}{\delta} = \varepsilon \quad (2)$$

Orientation angles and time—

$$V - \dot{\delta} = \dot{\varepsilon} \quad (3)$$

Orbital parameters and C_{nm}, S_{nm}—

$$V - \overset{\circ}{\delta} = \overset{\circ}{\varepsilon}$$

Object space coordinates—

$$V - \ddot{\delta} = \ddot{\varepsilon}$$

Altimeter distance measurement—

$$\hat{V} + \ddot{B}\ddot{\delta} + \overset{\circ\circ}{B}\overset{\circ\circ}{\delta} + \overset{\cdots\cdot}{B}\overset{\cdots\cdot}{\delta} = \hat{\varepsilon}. \quad (4)$$

$\dot{B}$ can be zero depending on the form of Equation 4. The V's are the residual vectors of the observed values, $\dot{\delta}$ are the correction vectors for the orientation angles and time, the B's represent the partial differentials with respect to the corresponding parameters, $\overset{\circ}{\delta}$ are the correction vectors for the orbital elements and C_{nm}, S_{nm}, $\ddot{\delta}$ are the correction vectors for the object point coordinates, and ε represents the discrepancy term arising from the measured value minus the approximations of the unknown parameters.

Grossly simplifying the problem, we write the entire merged set as

$$V + B\delta = \varepsilon. \quad (5)$$

Then the normal equations, according to the least squares criterion, are:

$$[B^tWB] = [B^tW\varepsilon]. \quad (6)$$

W is a weight matrix for all observed quantities. Comparing Equations 1 and 3, it is easy to see the difference in the concept of the geometrical MUSAT and the dynamical LOSAT/LOBAT systems. MUSAT does not impose orbital constraints.

In addition to the aforementioned conditions, both the MUSAT and LOSAT programs have scale and azimuth constraints.* The Apollo version of LOSAT/LOBAT has an expanded gravity model from 7,0 to 7,7 and will provide capability to recover center-of-mass coordinates, and to expand the ±1 radian of arc to ±2 radians of arc for orbital constraints. Plans are also underway to include a point-mass model to model more appropriately the much publicized lunar mass concentrations (Mascons). A simple layer model (onion-like layers) is being considered by some scientists as a means to represent the lunar potential.

These two programs offer unique capability for strip and block triangulations, and the computation of a lunar control point network given in ϕ, λ, h for each measured image. However, one more iteration is recommended for producing refinements in the spacecraft ephemeris and the gravity model coefficients, and for putting these refinements back into the control point network. This is possible with the proposed TRACE 8 Apollo

* Constraints are based on ground control points.

Program. The concept is to iterate between the photogrammetric programs and TRACE thereby taking advantage of their unique rigor and flexibility to effect improvements in the control point network.

TRACE is primarily an orbit ephemeris program that is being planned for expansion to accept photogrammetric image-point measurements and laser-altimeter distances. Past experience with the Lunar Orbiter's ephemerides strongly indicates that a means to tie the adjacent photo passes together is needed and that common photo points between adjacent passes is the means to do it. Of course, this is exactly what MUSAT and LOSAT do, but they do not have the long-arc multi-revolution, and gravity modeling capability as does TRACE. On the other hand, TRACE does not adjust camera attitudes and apply rigorous photogrammetric principles such as is done by MUSAT and LOSAT. So both programs are needed to work the problem iteratively.

The original TRACE Orbit Determination Program was written by the Aerospace Corporation.[14] The modified version has been recommended to NASA by the Apollo Photo Team. It should include the additional characteristics shown in Table 2.

Utilizing the data as anticipated from the three Apollo missions which will include altimeter distances on the back side in combination with the tracking data from Lunar Orbiter's polar Missions IV and V, (Figure 8) it seems reasonable to expect improvements in man's knowledge of the moon's physical characteristics. The polar orbits from Lunar Orbiter which occurred in 1966–67 can help define the moon's flattening and will also provide libration motion effects spread over a longer time frame than Apollo by itself. Although the theory seems straightforward, it has been pointed out by Doyle[15] (Team Chairman) that practical application is far from straightforward. Many quantities to be determined are strongly correlated: some are short-term, the others are long-term, some terms are better defined by polar orbits, others from inclined or equatorial orbits. To accomplish the complete job, one needs full photo coverage from different time, inclination, and altitudes which is not possible with the present plans. In any event, considerable experimentation will be needed to determine how far the solution can be pressed before the answers become meaningless.

GEOMETRIC REFERENCE SURFACE

After a dense, well distributed set of lunar

TABLE 2

Conditions Modeled	*Observation Data Type*
Disk Gravity Model	Altimeter Distance
Libration Motion	Photogrammetric Image Points
Center of Gravity Offset	Moon Centered Coordinates of Camera Station at time *t*.
Altimeter Terminus	Landmark Object Space Coordinates
Point Constraints	

ground feature coordinates X, Y, Z is computed from the triangulation, a mathematical reference surface can be determined. A surface-fitting procedure could be employed to determine the reference surface for the moon. Either a sphere, spheroid, or triaxial ellipsoid could be fitted by minimizing the sum of squares of differences between the triangulated topographic points and the selected reference surface equations of the general form.

$$Q = X^2/a^2 + Y^2/b^2 + Z^2/c^2 - 1 = 0$$

where Q is an oblate spheroid if $a=b$, Q is a sphere if $a=b=c$, Q is an ellipsoid if $a \neq b \neq c$, The best fitting Q should be adopted. The reference surface X, Y-plane would coincide with the moon's equator and its Z-axis would coincide with the axis of rotation. Its center would be the center of figure. It would be desirable to reduce the radius arbitrarily to make all topographic elements positive and perhaps use one of the Apollo lunar modules (with retro-reflector) now sitting on the moon as the Moon's Meades Ranch, datum point. Ranging retro-reflectors are located at the sites of Apollos 11, 14 and 15 and they should be identifiable on the photographs.

Current ranging accuracies (Earth to Moon) are on the order of ± 15 cm, and this accuracy will be reflected in the accuracy of the three point positions. Figure 1 shows how these points form a triangle which could serve as an absolute datum for the lunar reference surface. Also, it should be noted from Figure 1 that the points are contained within $\pm 26°$ latitude. This would degrade the validity of the reference surface at latitudes outside this area.

Up to now we have discussed the triangulation problem and the methods for establishing the control point network and a reference surface. The remaining task is to discuss the potential of the mapping and the pan camera system for producing topographic maps and information.

TABLE 3. MAP SCALES COMPATIBLE WITH RESOLUTION OF APOLLO PHOTOGRAPHY

Photography	S_p	r_p *lp/mm*	S_m *Optimum*	S_m *Operational*
Mapping Camera	1,500,000	58–70	210,000	250,000
Pan Camera Rectified	100,000	50	20,000	20,000

TABLE 4. ALLOWABLE PLANIMETRIC ERRORS

S_m	σ_p (*Class A*)	σ_p (*Class B*)
1,000,000	300 m	600 m
500,000	150	300
250,000	75	150
50,000	15	30
20,000	6	12

MAP COMPILATION POTENTIAL

Cartographic products contain three kinds of information:

* Content—details and surface features represented on the map.
* Position—location of features within the reference system.
* Elevation—spot heights and contour lines.

The potential of a mapping system can be determined by how it satisfies these three items. The Chairman of the Apollo Photo Team has evaluated the cameras in the SIM against these items and the essence of the results are presented below.

MAP CONTENT

It is usually accepted that the unaided human eye can resolve 5 to 10 line-pairs per millimeter (lp/mm). If we allow some safety factor for losses due to processing, we select 10 lp/mm and arrive at an expression for the optimum scale number at which photographic map products should be printed:

$$S_m = S_p \times 10/r_p$$

where S_m is the map scale number, S_p the photo scale number and r_p the photo resolution (lp/mm).

Table 3 shows the results of applying this criterion to the SIM photography.

Thus the recommended scales for reproducing the Apollo photography are 1:250,000 for the MCS and 1:20,000 for the rectified panoramic. Smaller scales will not fully exploit the information contained in the imagery.

POSITION

U.S. Map Accuracy Standards specify that the standard error of point positions σ_p for Class A maps should not exceed 0.3 mm at the scale of the published map:

$$\sigma_p(\text{meters}) = 3 \times 10^{-4} S_m.$$

For Class B maps the allowable errors are doubled. Table 4 lists the allowable planimetric positional errors for the various map scales.

The accuracy with which positions can be established photogrammetrically depends upon the image scale and the accuracy of measurement on the photographs.

$$\sigma_p(\text{meters}) = S_p\sigma_x$$

where σ_x the standard error of image coordinate measurement in meters. Experience has shown that analytical treatment of frame photography usually results in $\sigma_x \simeq 10$ μm, whereas map compilation using $2\times$ enlargements of the frame photographs in standard projection-type stereoplotters, may have $\sigma_x \simeq 100$ μm at the enlarged photo scale. The planimetric accuracy σ_p, and corresponding map scales obtainable with the Apollo MCS (frame photography) are given in Table 5.

Because of its inherently poor geometrical quality, panoramic photography is not well accepted for establishing planimetric positions. Table 4 shows that direct compilation of frame photographs can produce Class A planimetric positions at the 1:250,000 scale which is compatible with the frame photographic resolution. At the 1:25,000 scale, which is roughly compatible with the resolution of the rectified panoramic photography, the planimetric accuracy obtained will be adequate for Class B standards. The criterion above is based on what the unaided eye can see and on national standards. There is no fundamental reason why photo image maps at larger scales cannot be compiled. Enlargements of Lunar Orbiter photographs until the resolution is perhaps only 1 lp/mm has proven useful for landing-site maps and data packages. These larger scales are particularly useful when the interpreter needs a lot of room for annotating names, symbols, and grids to plot the geology and the geological traverses planned for the astronauts.

ELEVATIONS AND CONTOURING

A statistical interpretation of U.S. Map Accuracy Standards defines the relation between the standard error of elevation and the contour interval;

$$\sigma_h = 0.3\ c.i.\ (\text{Class } A)$$
$$= 0.6\ c.i.\ (\text{Class } B)$$

TABLE 5. MAP SCALES FOR APOLLO FRAME PHOTOGRAPHS

Technique	σ_p	S_m (*Class A*)	S_m (*Class B*)
Analytic Computation	15m	50,000	25,000
Projection Compilation	75m	250,000	125,000

where σ_h is the standard error of elevation and *c.i.* is the contour interval. There is a growing opinion among photogrammetrists that contours can be drawn at intervals nearly equal to the standard error in elevation and still meet national standards.

Elevation data is obtained from aerial photographs by virtue of image parallax which depends on the base-to-height ratio of the stereo model, the image scale and the measuring accuracy. The well known formula for standard error in elevation is useful in expressing the internal accuracy of a photogrammetrically compiled map:

$$\sigma_h = S_p(H/B)\sigma_x.$$

where H is the flight altitude, B is the distance between exposure stations of model and σ_x is the standard error of parallax measurement. The frame photographs will have 78 percent forward overlap between adjacent pictures. This provides a ratio $H/B=0.75$ for photos n and $n+4$.

As mentioned before, panoramic photography has inherently poor geometry, and hence the accuracy of contour mapping depends on elevation points determined by triangulating the MCS frame photography. The plan is to fit the high-resolution panoramic stereo models as shown in Figure 6, to the common image points established by the frame triangulation. However, due to pan photography being larger scale and higher resolution, it is possible to discriminate elevation differences with high precision. The 25° convergence provides a ratio $H/B=2.25$. With this pan configuration associated with the frame triangulation points for orientation, and then estimating σ_x for various stereo plotters, Table 6 summarizes the contouring potential of the systems.

SUMMARY

In summary, the Apollo MCS, panoramic system, and its associated data sensors over three missions, should provide the scientific community with a wealth of information. If exploited properly, the reference coordinate system and its related physical constants will be valuable to science and lunar navigation,

TABLE 6. CONTOURING CAPABILITY OF APOLLO PHOTOGRAPHY

Photography	σ_x	σ_h	*c.i.* (*Class A*)	*c.i.* (*Class B*)
Frame				
Triangulation	10μm	11m	40m	20m
Projection Plotter*	30	22	80	40
Precision Plotter	15	11	40	20
Panoramic				
Contouring	15	11	40	20
Form Lining	15	3.4	10	5

* 2× Frame Scale = 1:750,000

in addition to providing a basis for a sound mapping program between ±26° latitude.

The mapping program will provide a cartographic base to plan future lunar exploration and to display and record a variety of lunar scientific inferences for history.

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